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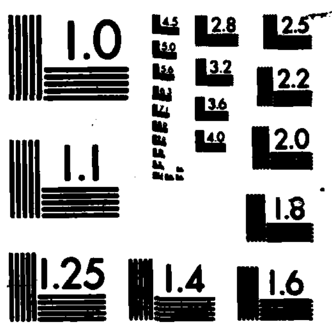
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NSWC MP 85-466

# UNIFORM TRANSVERSE MAGNETIC FIELD INSIDE A LONG CYLINDER

BY K. T. NGUYEN H. S. UHM

RESEARCH AND TECHNOLOGY DEPARTMENT

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For applications where uniform transverse magnetic field is required over a substantial distance, the use of dipole magnets can be quite cumbersome. In this illustrative article, we present a simple technique to achieve a uniform and uniaxial transverse magnetic field inside a long cylinder, which can be easily carried out.

# FOREWORD

For applications where uniform transverse magnetic field is required over a substantial distance, the use of dipole magnets can be quite cumbersome. In this illustrative article, we present a simple technique to achieve a uniform and uniaxial transverse magnetic field inside a long cylinder, which can be easily carried out.

Approved by:

H. R. RIEDL, Acting Head
Radiation Division

The steering or bending of charged particle beams has normally been accomplished by the use of a uniform magnetic field applied transversly to the plane of beam propagation. In the past this transverse magnetic field has been obtained by the use of dipole magnets. While this arrangement is advantageous when the applied field is required for only a short distance, as in beam steering, it is often cumbersome, for certain applications, when a uniform transverse magnetic field is required over a long distance.

In this article, we show that a uniform transverse magnetic field can be created inside a long cylinder of radius a with currents running along the tube distributed as $I(\theta) = I_m \sin\theta$, as shown in Figure 1. In this arrangement, it is demonstrated that the magnetic field inside the tube can be expressed as:

$$\vec{B} = \frac{\mu_0 I_m}{2a}\hat{y}. \tag{1}$$

To illustrate this fact, we start out with Ampere's law

$$\nabla \times \vec{B} = \mu_0 \vec{J}, \tag{2}$$

where $\vec{B} = \nabla \times \vec{A}$

$$\vec{J} = \frac{I_m}{a}\sin\theta\ \delta(r - a)\ \hat{z}.$$

Due to the symmetry of the problem at hand, it is obvious that the vector potential $\vec{A}$ has only the $\hat{z}$ component and is independent of z, i.e.

$$\vec{A} = \hat{z}\, A_z(r,\theta). \tag{3}$$

Therefore, the Ampere's law can be written in cylindrical coordinates as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 A_z}{\partial \theta^2} = \frac{\mu_0 I_m}{a}\sin\theta\ \delta(r - a). \tag{4}$$

For $r \neq a$, this equation can readily be solved by separation of variables, i.e.

$$A_z = R(r)\ \Phi(\theta). \tag{5}$$

Substitute this in equation (4) we then obtain, except at $r = a$

$$\begin{aligned} &\frac{d^2\Phi}{d\theta^2} = -\, m^2\Phi, \\ &r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} - m^2R = 0, \qquad m = 0, \pm 1, \pm 2, \ldots \end{aligned} \tag{6}$$

These equations in turn give us the following solutions

$$\begin{aligned} \Phi(\theta) &= \begin{cases} \sin(m\theta), \\ \cos(m\theta), \end{cases} \\ R(r) &= \quad r^m. \end{aligned} \tag{7}$$

However, symmetry requires that $\Phi(\theta) = -\,\Phi(-\theta)$. As a result, we choose $\Phi(\theta) = \sin(m\theta)$ and the general form of $A_z$ can be written as

$$A_z(r,\theta) = \sum_{m=-\infty}^{\infty} b_m r^m \sin(m\theta), \tag{8}$$

where the coefficients $b_m$ are constant to be determined.

To complete the evaluation of the coefficients $b_m$, we separate the problem into 2 cases: inside of tube and outside of tube.

Inside of tube: In this case, since $A_z$ must be finite at $r = 0$, therefore we require $m > 0$. Then the vector potential $A_z^i$ inside the tube reduces to

$$A_z^i\ (r,\theta) = \sum_{m=0}^{\infty} b_m^i r^m \sin(m\theta). \tag{9}$$

The radial and angular components of the magnetic field can be written as

$$\begin{aligned} B_r^i(r,\theta) &= (\nabla \times \vec{A}^i)_r = \frac{1}{r}\frac{\partial A_z^i}{\partial \theta} = \sum_{m=1}^{\infty} m b_m^i\, r^{m-1} \cos(m\theta) \\ B_\theta^i(r,\theta) &= (\nabla \times \vec{A}^i)_\theta = -\frac{\partial A_z^i}{\partial r} = -\sum_{m=1}^{\infty} m b_m^i\, r^{m-1} \sin(m\theta). \end{aligned} \tag{10}$$

In order to calculate the coefficients $b_m^i$, we set $\theta = 0$, and compare $B_r^i(r,\ \theta = 0)$ with $B_y^i\ (x = 0,y)$, which we are presently calculating.

The azimuthal field generated by an infinitely long wire with current I can be written as

$$B^*(r) = \frac{\mu_0 I}{2\pi r}. \tag{11}$$

This fact, together with Figure 2, gives us the physical basis for the following equation:

$$dB_y^i(0,y) = \frac{\mu_0 I(\theta) d\theta}{2\pi} \times \frac{a \sin\theta}{a^2 + y^2 - 2ay\cos\theta}, \tag{12}$$

where $dB_y^i(0,y)$ is the contribution to the $B_y^i$ field at $(x,y) = (0,y)$ on the equatorial plane from the current element $I(\theta)$. Since $I(\theta) = I_m \sin\theta$, we can therefore write

$$B_y^i(0,y) = \frac{\mu_0 I_m}{2\pi} \int_0^{2\pi} \frac{a\sin^2\theta \, d\theta}{a^2 + y^2 - 2ay\cos\theta}. \quad (13)$$

Using a change of variable $z = \cos\theta$, and defining $p = -(a^2 + y^2)/2ay$, then the integral in equation (13) can be rewritten as

$$B_y^i(0,y) = -\frac{\mu_0 I_m}{2\pi y} \int_{-1}^{1} \frac{1 - z^2}{z + p} dz. \quad (14)$$

After straightforward algebra, the above integral can be found to give the following result

$$B_y^i(0,y) = -\frac{\mu_0 I_m}{2y} \left[p + (p^2 - 1)^{1/2}\right] = \frac{\mu_0 I_m}{2a}. \quad (15)$$

Comparing $B_r^i(r = y, \theta = 0)$ with $B_y^i(0,y)$, we finally obtain

$$b_m^i = \begin{cases} \dfrac{\mu_0 I_m}{2a} & \text{for } m = 1, \\ 0 & \text{for } m > 2. \end{cases} \quad (16)$$

Thus, the cylindrical components of the magnetic field inside the tube can readily be written as

$$B_r^i(r,\theta) = \frac{\mu_0 I_m}{2a} \cos\theta,$$
$$B_\theta^i(r,\theta) = -\frac{\mu_0 I_m}{2a} \sin\theta. \quad (17)$$

Equivalently, the magnetic field inside the tube can be expressed in Cartesian coordinates as:

$$B_x^i(x,y) = B_r \sin\theta + B_\theta \cos\theta = 0,$$
$$B_y^i(x,y) = B_r \cos\theta - B_\theta \sin\theta = \frac{\mu_0 I_m}{2a}, \tag{18}$$

which clearly shows that the field inside the cylinder is uniform and uniaxial as we expect

<u>Outside of tube</u>: In this case, the vector potential can be written as

$$A_z^o(r,\theta) = \sum_{n=1}^{\infty} b_n^o \frac{\sin(n\theta)}{r^n}, \tag{19}$$

so that $A_z^o \to 0$ as $r \to \infty$. In order to evaluate the coefficient $c_n$, we use the appropriate boundary conditions at $r = a$, that is $A_z$ must be continuous at $r = a$, i.e.

$$A_z^o(a,\theta) = A_z^i(a,\theta),$$

or

$$\sum_{n=1}^{\infty} b_n^o \frac{\sin(n\theta)}{a^n} = \frac{\mu_0 I_m}{2} \sin\theta. \tag{20}$$

From the above equation, we obtain:

$$b_n^o = \begin{cases} \frac{\mu_0 I_m}{2} & \text{for } n = 1, \\ 0 & \text{for } n > 2, \end{cases} \tag{21}$$

and the vector potential can now be explicitly written as

$$A(r,\theta) = \begin{cases} \dfrac{\mu_0 I_m}{2a} r\sin\theta & \text{for } r < a, \\ \dfrac{\mu_0 I_m a}{2r} \sin\theta & \text{for } r > a. \end{cases} \tag{22}$$

It is important to note here that the vector potential above also satisfies the jump condition at $r = a$, i.e.

$$\left(\frac{\partial A_z}{\partial r}\right)_{r=a^+} - \left(\frac{\partial A_z}{\partial r}\right)_{r=a^-} = \frac{\mu_0 I_m}{a} \sin\theta, \tag{23}$$

which has been obtained by integrating equation (4) across the boundary.

The magnetic field outside the tube can now be expressed in cylindrical coordinates as:

$$\begin{aligned} B_r^o(r,\theta) &= \frac{1}{r}\frac{\partial A_z^o}{\partial \theta} = \frac{\mu_0 I_m a}{2r^2}\cos\theta, \\ B_\theta^o(r,\theta) &= -\frac{\partial A_z^o}{\partial r} = \frac{\mu_0 I_m a}{2r^2}\sin\theta, \end{aligned} \tag{24}$$

or equivalently in terms of Cartesian coordinates as:

$$\begin{aligned} B_x^o(x,y) &= \mu_0 I_m a \frac{xy}{(x^2+y^2)^2}, \\ B_y^o(x,y) &= \frac{\mu_0 I_m a}{2} \frac{x^2 - y^2}{(x^2+y^2)^2}. \end{aligned} \tag{25}$$

Equations (18) and (24) form the main result of this article, which clearly shows that a uniform transverse magnetic field can be created inside a long cylinder, if the current wires running axially along the tube are distributed as $I(x,y) = I_m x/a$.

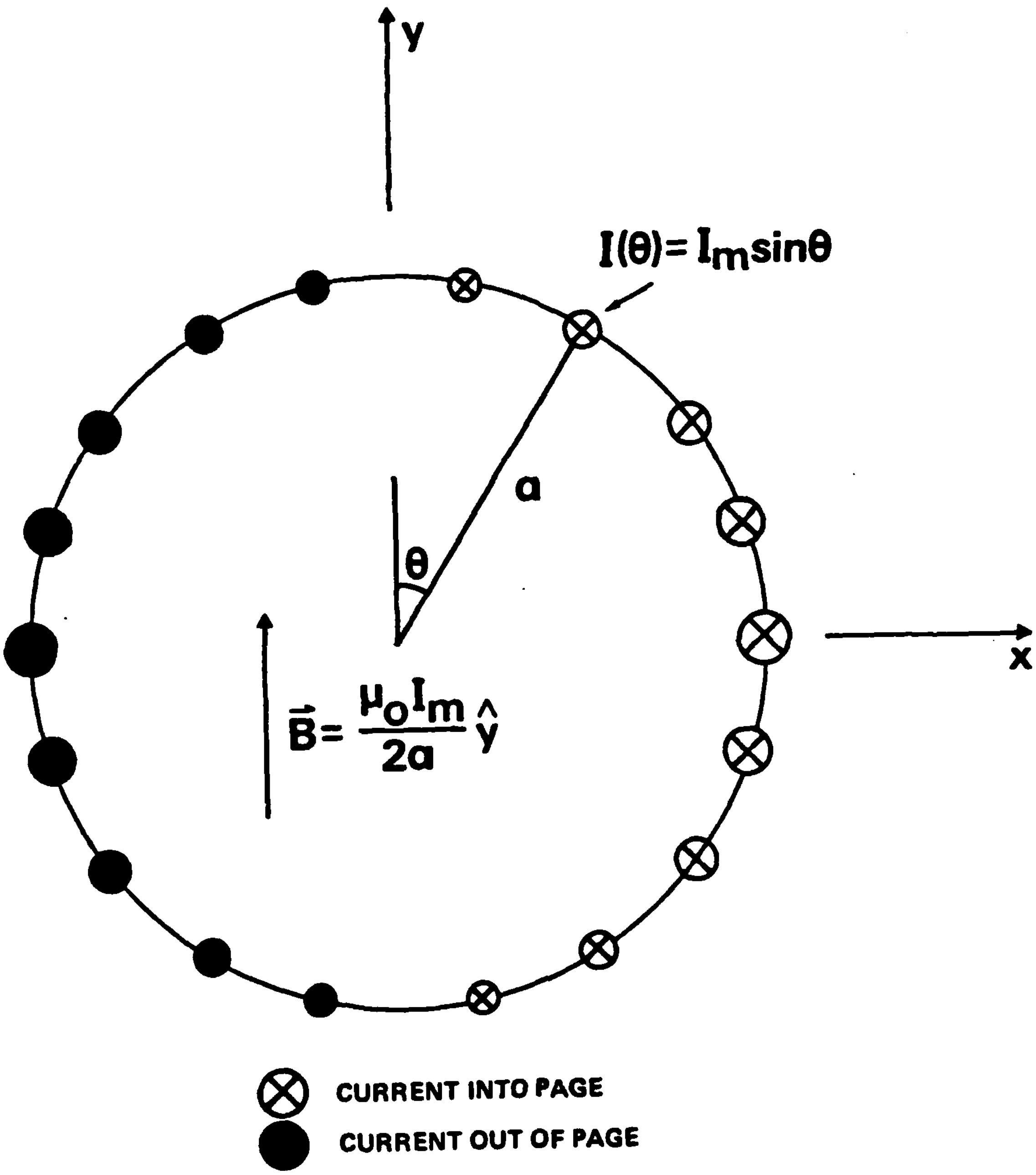


FIGURE 1. GEOMETRY OF THE SETUP WITH CURRENTS RUNNING ALONG Z DIRECTION

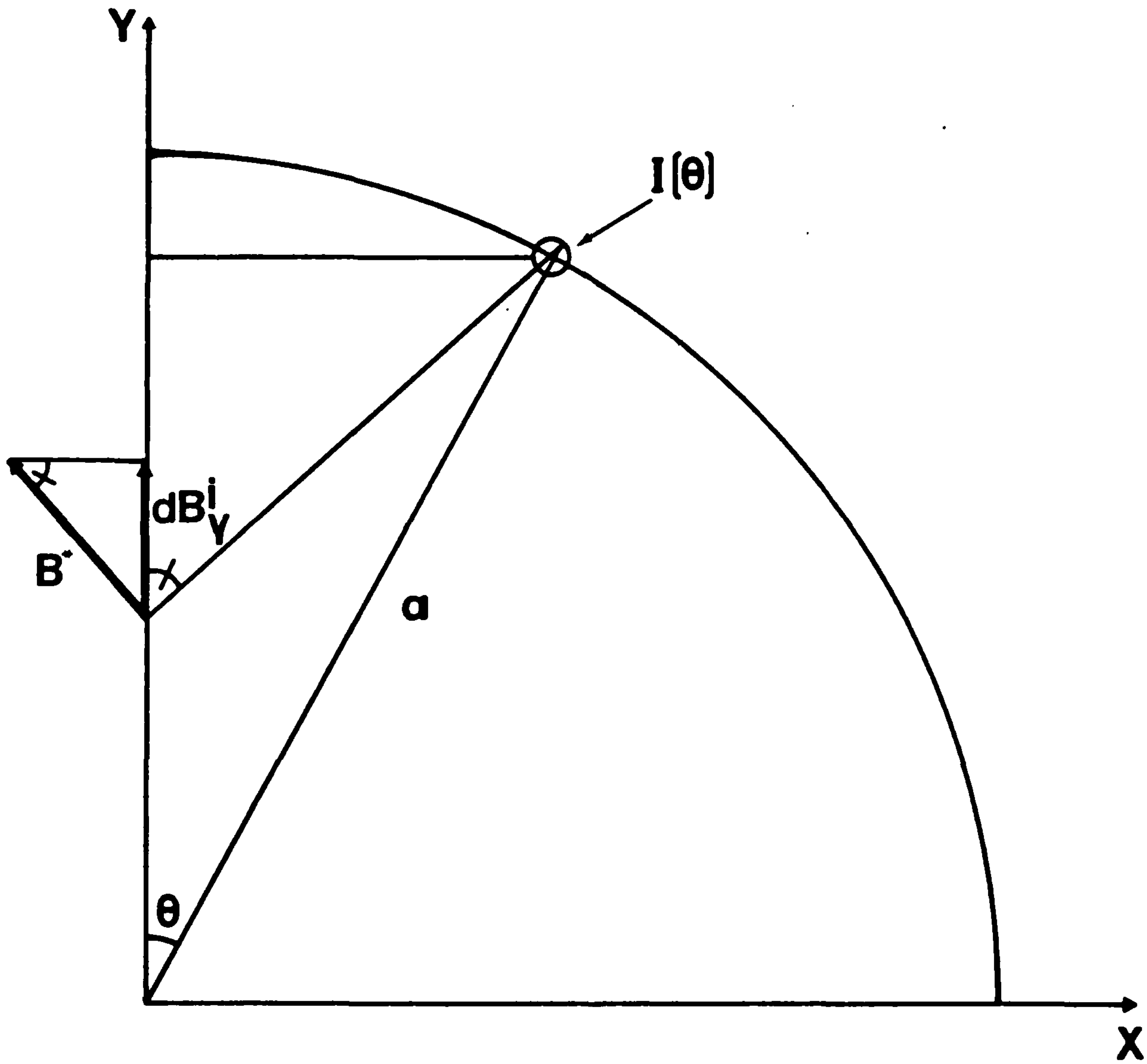


FIGURE 2. CONTRIBUTION TO $B_y^i$ (o, y) FROM CURRENT ELEMENT I(θ)

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